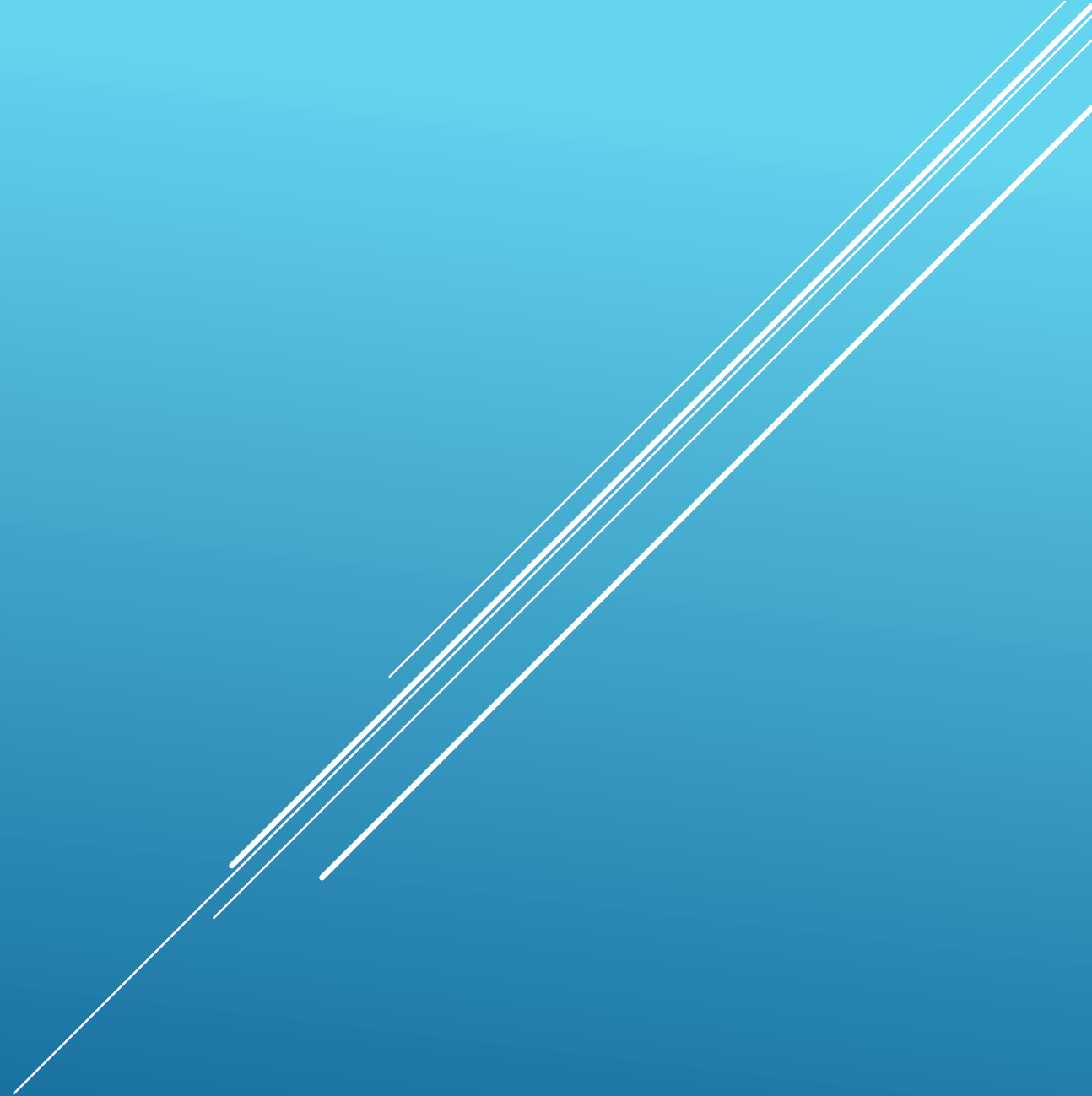


# PERTEMUAN 1




- ▶ Tipe dan tahapan dalam pengambilan keputusan
- ▶ Pengambilan keputusan dalam ketidakpastian
- ▶ Pengambilan keputusan dalam resiko
- ▶ Decision Trees
- ▶ Teori utilitas

# TEORI PENGAMBILAN KEPUTUSAN

# WHAT IS QUANTITATIVE ANALYSIS?

*Quantitative analysis* is a scientific approach to managerial decision making whereby raw data are processed and manipulated resulting in meaningful information



- ▶ *Quantitative analysis uses a scientific approach to decision making.*
  - ▶ *Both qualitative and quantitative factors must be considered*
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# THE QUANTITATIVE ANALYSIS APPROACH

*The types of models include physical, scale, schematic, and mathematical models.*



Figure 1.1

- An important part of the quantitative analysis approach
- Let's look at a simple mathematical model of profit

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

HOW TO DEVELOP A QUANTITATIVE  
ANALYSIS MODEL

Expenses can be represented as the sum of fixed and variable costs and variable costs are the product of unit costs times the number of units

$$\text{Profit} = \text{Revenue} - (\text{Fixed cost} + \text{Variable cost})$$

$$\text{Profit} = (\text{Selling price per unit})(\text{number of units sold}) - [\text{Fixed cost} + (\text{Variable costs per unit})(\text{Number of units sold})]$$

$$\text{Profit} = sX - [f + vX]$$

$$\text{Profit} = sX - f - vX$$

The *parameters* of this model are  $f$ ,  $v$ , and  $s$  as these are the inputs inherent in the model

The *decision variable* of interest is  $X$

where

$s$  = selling price per unit

$f$  = fixed cost


$v$  = variable cost per unit

$X$  = number of units sold

# MODELS CATEGORIZED BY RISK

- ▶ Mathematical models that do not involve risk are called *deterministic* models
  - ▶ We know all the values used in the model with complete certainty
- ▶ Mathematical models that involve risk, chance, or uncertainty are called *probabilistic* models
  - ▶ Values used in the model are estimates based on probabilities



- ▶ What is involved in making a good decision?
  - ▶ Decision theory is an analytic and systematic approach to the study of decision making
  - ▶ A good decision is one that is based on logic, considers all available data and possible alternatives, and the quantitative approach described here
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# TYPES OF DECISION-MAKING ENVIRONMENTS

Type 1: Decision making under certainty

- ▶ Decision maker *knows with certainty* the consequences of every alternative or decision choice

Type 2: Decision making under uncertainty


- ▶ The decision maker *does not know* the probabilities of the various outcomes

Type 3: Decision making under risk

- ▶ The decision maker *knows the probabilities* of the various outcomes

# DECISION MAKING UNDER UNCERTAINTY

**There are several criteria for making decisions under uncertainty**

1. Maximax (optimistic)
  2. Maximin (pessimistic)
  3. Criterion of realism (Hurwicz)
  4. Equally likely (Laplace)
  5. Minimax regret
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# MAXIMAX

Used to find the alternative that maximizes the maximum payoff

- Locate the maximum payoff for each alternative
- Select the alternative with the maximum number

| ALTERNATIVE             | STATE OF NATURE       |                         | MAXIMUM IN A ROW (\$) |
|-------------------------|-----------------------|-------------------------|-----------------------|
|                         | FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |                       |
| Construct a large plant | 200,000               | -180,000                | 200,000               |
| Construct a small plant | 100,000               | -20,000                 | 100,000               |
| Do nothing              | 0                     | 0                       | 0                     |

Table 3.2

# MAXIMIN

Used to find the alternative that maximizes the minimum payoff

- Locate the minimum payoff for each alternative
- Select the alternative with the maximum number

| ALTERNATIVE             | STATE OF NATURE       |                         | MINIMUM IN A ROW (\$) |
|-------------------------|-----------------------|-------------------------|-----------------------|
|                         | FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |                       |
| Construct a large plant | 200,000               | -180,000                | -180,000              |
| Construct a small plant | 100,000               | -20,000                 | -20,000               |
| Do nothing              | 0                     | 0                       | 0                     |

Maximin

Table 3.3

# CRITERION OF REALISM (HURWICZ)

A *weighted average* compromise between optimistic and pessimistic

- Select a coefficient of realism  $\alpha$
- Coefficient is between 0 and 1
- A value of 1 is 100% optimistic
- Compute the weighted averages for each alternative
- Select the alternative with the highest value

$$\text{Weighted average} = \alpha(\text{maximum in row}) + (1 - \alpha)(\text{minimum in row})$$

# CRITERION OF REALISM (HURWICZ)

- For the large plant alternative using  $\alpha = 0.8$   
 $(0.8)(200,000) + (1 - 0.8)(-180,000) = 124,000$
- For the small plant alternative using  $\alpha = 0.8$   
 $(0.8)(100,000) + (1 - 0.8)(-20,000) = 76,000$

| ALTERNATIVE             | STATE OF NATURE       |                         | CRITERION OF REALISM<br>( $\alpha = 0.8$ )\$ |
|-------------------------|-----------------------|-------------------------|--|
|                         | FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |  |
| Construct a large plant | 200,000               | -180,000                | 124,000                                      |
| Construct a small plant | 100,000               | -20,000                 | 76,000                                       |
| Do nothing              | 0                     | 0                       | 0  |

Table 3.4

# EQUALLY LIKELY (LAPLACE)

**Considers all the payoffs for each alternative**

- Find the average payoff for each alternative
- Select the alternative with the highest average

| ALTERNATIVE                | STATE OF NATURE          |                            | ROW<br>AVERAGE (\$) |
|----------------------------|--------------------------|----------------------------|---------------------|
|                            | FAVORABLE<br>MARKET (\$) | UNFAVORABLE<br>MARKET (\$) |                     |
| Construct a large<br>plant | 200,000                  | -180,000                   | 10,000              |
| Construct a small<br>plant | 100,000                  | -20,000                    | 40,000              |
| Do nothing                 | 0                        | 0                          | 0                   |

Equally likely

Table 3.5



# MINIMAX REGRET

Based on *opportunity loss* or *regret*, the difference between the optimal profit and actual payoff for a decision

- ▶ Create an opportunity loss table by determining the opportunity loss for not choosing the best alternative
- ▶ Opportunity loss is calculated by subtracting each payoff in the column from the best payoff in the column
- ▶ Find the maximum opportunity loss for each alternative and pick the alternative with the minimum number

# MINIMAX REGRET

## ■ Opportunity Loss Tables

| STATE OF NATURE       |                         |
|-----------------------|-------------------------|
| FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |
| 200,000 – 200,000     | 0 – (–180,000)          |
| 200,000 – 100,000     | 0 – (–20,000)           |
| 200,000 – 0           | 0 – 0                   |

Table 3.6

| ALTERNATIVE             | STATE OF NATURE       |                         |
|-------------------------|-----------------------|-------------------------|
|                         | FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |
| Construct a large plant | 0                     | 180,000                 |
| Construct a small plant | 100,000               | 20,000                  |
| Do nothing              | 200,000               | 0                       |

Table 3.7

# MINIMAX REGRET

| ALTERNATIVE                | STATE OF NATURE          |                            | MAXIMUM IN<br>A ROW (\$) |
|----------------------------|--------------------------|----------------------------|--------------------------|
|                            | FAVORABLE<br>MARKET (\$) | UNFAVORABLE<br>MARKET (\$) |                          |
| Construct a large<br>plant | 0                        | 180,000                    | 180,000                  |
| Construct a small<br>plant | 100,000                  | 20,000                     | 100,000                  |
| Do nothing                 | 200,000                  | 0                          | 200,000                  |

Minimax

Table 3.8

# DECISION MAKING UNDER RISK

- ▶ Decision making when there are several possible states of nature and we know the probabilities associated with each possible state
- ▶ Most popular method is to choose the alternative with the highest *expected monetary value (EMV)*

$$\begin{aligned} \text{EMV (alternative } i) = & (\text{payoff of first state of nature}) \\ & \times (\text{probability of first state of nature}) \\ & + (\text{payoff of second state of nature}) \\ & \times (\text{probability of second state of nature}) \\ & + \dots + (\text{payoff of last state of nature}) \\ & \times (\text{probability of last state of nature}) \end{aligned}$$

# EMV FOR THOMPSON LUMBER

- Each market has a probability of 0.50
- Which alternative would give the highest EMV?
- The calculations are

$$\begin{aligned}\text{EMV (large plant)} &= (0.50)(\$200,000) + (0.50)(-\$180,000) \\ &= \$10,000\end{aligned}$$

$$\begin{aligned}\text{EMV (small plant)} &= (0.50)(\$100,000) + (0.50)(-\$20,000) \\ &= \$40,000\end{aligned}$$

$$\begin{aligned}\text{EMV (do nothing)} &= (0.50)(\$0) + (0.50)(\$0) \\ &= \$0\end{aligned}$$

# EMV FOR THOMPSON LUMBER

| ALTERNATIVE                | STATE OF NATURE          |                            | EMV (\$) |
|----------------------------|--------------------------|----------------------------|----------|
|                            | FAVORABLE<br>MARKET (\$) | UNFAVORABLE<br>MARKET (\$) |          |
| Construct a large<br>plant | 200,000                  | -180,000                   | 10,000   |
| Construct a small<br>plant | 100,000                  | -20,000                    | 40,000   |
| Do nothing                 | 0                        | 0                          | 0        |
| Probabilities              | 0.50                     | 0.50                       |          |

Table 3.9

Largest EMV

# EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

- ▶ EVPI places an upper bound on what you should pay for additional information


$$EVPI = EV_{wPI} - \text{Maximum EMV}$$

*EVPI is the increase in EMV that results from having perfect information*

- ▶  $EV_{wPI}$  is the long run average return if we have perfect information before a decision is made
  - ▶ *We compute the best payoff for each state of nature since we don't know, until after we pay, what the research will tell us*

$$\begin{aligned} EV_{wPI} = & (\text{best payoff for first state of nature}) \\ & \times (\text{probability of first state of nature}) \\ & + (\text{best payoff for second state of nature}) \\ & \times (\text{probability of second state of nature}) \\ & + \dots + (\text{best payoff for last state of nature}) \\ & \times (\text{probability of last state of nature}) \end{aligned}$$

# EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

- ▶ Scientific Marketing, Inc. offers analysis that will provide certainty about market conditions (favorable)
  - ▶ Additional information will cost \$65,000
  - ▶ Is it worth purchasing the information?
- 
- Three parallel white lines of varying lengths are positioned in the bottom right corner of the slide, slanted diagonally upwards from left to right.



# EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

1. **Best alternative for favorable state of nature is build a large plant with a payoff of \$200,000**  
**Best alternative for unfavorable state of nature is to do nothing with a payoff of \$0**

$$EV_{wPI} = (\$200,000)(0.50) + (\$0)(0.50) = \$100,000$$

2. **The maximum EMV without additional information is \$40,000**

$$\begin{aligned} EVPI &= EV_{wPI} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000 \end{aligned}$$

## EXPECTED VALUE OF PERFECT INFORMATION (EVPI)

1. Best alternative for favorable state of nature is build a large plant with a payoff of \$200,000  
Best alternative for unfavorable state of nature is to do nothing with a payoff of \$0

$$EV_{wPI} = (\$200,000)(0.50) + (\$0)(0.50) = \$100,000$$

2. The maximum EMV without additional information is \$40,000

$$\begin{aligned} EVPI &= EV_{wPI} - \text{Maximum EMV} \\ &= \$100,000 - \$40,000 \\ &= \$60,000 \end{aligned}$$

So the maximum Thompson should pay for the additional information is \$60,000

# EVWPI

| <i>Alternative</i>      | <b>State of Nature</b>       |                                | <i>EMV</i>      |
|-------------------------|------------------------------|--------------------------------|-----------------|
|                         | <i>Favorable Market (\$)</i> | <i>Unfavorable Market (\$)</i> |                 |
| Construct a large plant | 200,000                      | -180,000                       | 10,000          |
| Construct a small plant | 100,000                      | -20,000                        | 40,000          |
| Do nothing              | 0                            | 0                              | 0               |
| Perfect Information     | 200,000                      | 0                              | EVwPI = 100,000 |

## ■ Compute EVwPI

- The best alternative with a favorable market is to build a large plant with a payoff of \$200,000. In an unfavorable market the choice is to do nothing with a payoff of \$0
- $EVwPI = (\$200,000) \cdot .5 + (\$0) \cdot (.5) = \$100,000$

## ■ Compute EVPI = EVwPI – max EMV = \$100,000 - \$40,000 = \$60,000

- The most we should pay for any information is \$60,000

# IN-CLASS EXAMPLE

- Using the table below compute EMV,  $EV_{wPI}$ , and EVPI.

| Alternative           | State of Nature  |                     |                  |
|-----------------------|------------------|---------------------|------------------|
|                       | Good Market (\$) | Average Market (\$) | Poor Market (\$) |
| Construct large plant | 75,000           | 25,000              | -40,000          |
| Construct small plant | 100,000          | 35,000              | -60,000          |
| Do nothing            | 0                | 0                   | 0                |
| Probability           | 0.25             | 0.50                | 0.25             |

# IN-CLASS EXAMPLE: EMV AND EVWPI SOLUTION

| Alternative           | State of Nature  |                     |                  | EMV    |
|-----------------------|------------------|---------------------|------------------|--------|
|                       | Good Market (\$) | Average Market (\$) | Poor Market (\$) |        |
| Construct large plant | 75,000           | 25,000              | -40,000          | 21,250 |
| Construct small plant | 100,000          | 35,000              | -60,000          | 27,500 |
| Do nothing            | 0                | 0                   | 0                | 0      |
| Probability           | 0.25             | 0.50                | 0.25             |        |

$$EVPI = EVwPI - \max(EMV)$$

$$\begin{aligned} EVwPI &= \$100,000*0.25 + \$35,000*0.50 + 0*0.25 \\ &= \$42,500 \end{aligned}$$

$$\begin{aligned} EVPI &= \$42,500 - \$27,500 \\ &= \$15,000 \end{aligned}$$

IN-CLASS EXAMPLE:  
EVPI SOLUTION

# EXPECTED OPPORTUNITY LOSS

- ▶ *Expected opportunity loss* (EOL) is the cost of not picking the best solution
- ▶ First construct an opportunity loss table
- ▶ For each alternative, multiply the opportunity loss by the probability of that loss for each possible outcome and add these together
- ▶ Minimum EOL will always result in the same decision as maximum EMV
- ▶ Minimum EOL will always equal EVPI

# THOMPSON LUMBER: PAYOFF TABLE

| Alternative             | State of Nature       |                         |
|-------------------------|-----------------------|-------------------------|
|                         | Favorable Market (\$) | Unfavorable Market (\$) |
| Construct a large plant | 200,000               | -180,000                |
| Construct a small plant | 100,000               | -20,000                 |
| Do nothing              | 0                     | 0                       |
| Probabilities           | 0.50                  | 0.50                    |



# THOMPSON LUMBER: EOL

## THE OPPORTUNITY LOSS TABLE

| ALTERNATIVE             | STATE OF NATURE          |                            | EOL     |
|-------------------------|--------------------------|----------------------------|---------|
|                         | FAVORABLE<br>MARKET (\$) | UNFAVORABLE<br>MARKET (\$) |         |
| Construct a large plant | 200,000 -<br>200,000     | 0-(-180,000)               | 90,000  |
| Construct a small plant | 200,000 -<br>100,000     | 0-(-20,000)                | 60,000  |
| Do nothing              | 200,000 - 0              | 0-0                        | 100,000 |
| Probabilities           | 0.50                     | 0.50                       |         |

| ALTERNATIVE             | STATE OF NATURE          |                            | EOL     |
|-------------------------|--------------------------|----------------------------|---------|
|                         | FAVORABLE<br>MARKET (\$) | UNFAVORABLE<br>MARKET (\$) |         |
| Construct a large plant | 0                        | 180,000                    | 90,000  |
| Construct a small plant | 100,000                  | 20,000                     | 60,000  |
| Do nothing              | 200,000                  | 0                          | 100,000 |
| Probabilities           | 0.50                     | 0.50                       |         |

## THOMPSON LUMBER: OPPORTUNITY LOSS TABLE

# EXPECTED OPPORTUNITY LOSS

| ALTERNATIVE             | STATE OF NATURE       |                         | EOL     |
|-------------------------|-----------------------|-------------------------|---------|
|                         | FAVORABLE MARKET (\$) | UNFAVORABLE MARKET (\$) |         |
| Construct a large plant | 0                     | 180,000                 | 90,000  |
| Construct a small plant | 100,000               | 20,000                  | 60,000  |
| Do nothing              | 200,000               | 0                       | 100,000 |
| Probabilities           | 0.50                  | 0.50                    |         |

Table 3.10

Minimum EOL

$$\text{EOL (large plant)} = (0.50)(\$0) + (0.50)(\$180,000) = \$90,000$$

$$\text{EOL (small plant)} = (0.50)(\$100,000) + (0.50)(\$20,000) = \$60,000$$

$$\text{EOL (do nothing)} = (0.50)(\$200,000) + (0.50)(\$0) = \$100,000$$

EOL

- ▶ The minimum EOL will always result in the same decision (NOT value) as the maximum EMV
- ▶ The EVPI will always equal the minimum EOL

$$\text{EVPI} = \text{minimum EOL}$$

# SENSITIVITY ANALYSIS

- Sensitivity analysis examines how our decision might change with different input data
- For the Thompson Lumber example

$P$  = probability of a favorable market

$(1 - P)$  = probability of an unfavorable market

# SENSITIVITY ANALYSIS

$$\begin{aligned}\text{EMV}(\text{Large Plant}) &= \$200,000P - \$180,000(1 - P) \\ &= \$200,000P - \$180,000 + \$180,000P \\ &= \$380,000P - \$180,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{Small Plant}) &= \$100,000P - \$20,000(1 - P) \\ &= \$100,000P - \$20,000 + \$20,000P \\ &= \$120,000P - \$20,000\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{Do Nothing}) &= \$0P + 0(1 - P) \\ &= \$0\end{aligned}$$

# SENSITIVITY ANALYSIS

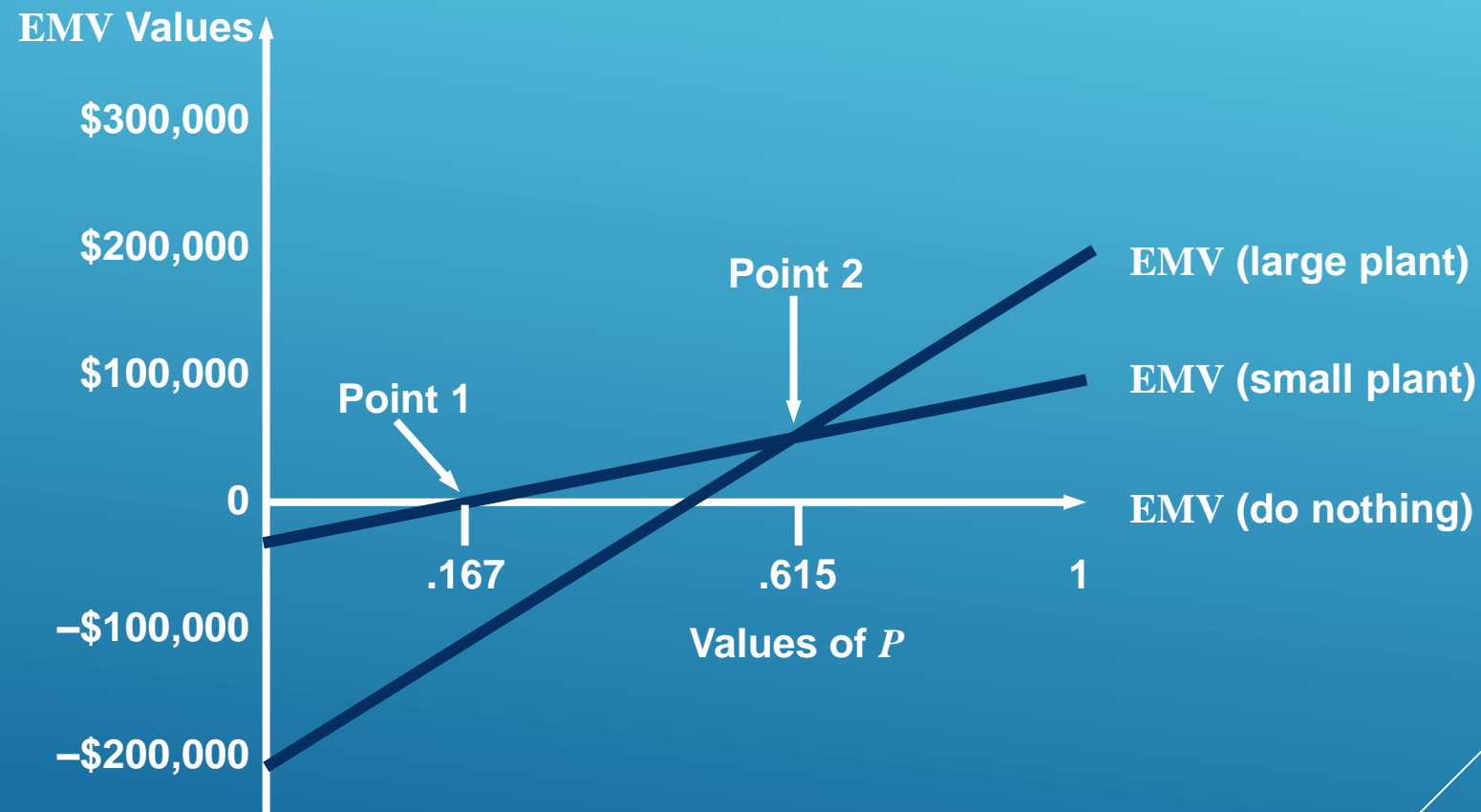


Figure 3.1

# SENSITIVITY ANALYSIS

Point 1:

EMV(do nothing) = EMV(small plant)

$$0 = \$120,000P - \$20,000(1-P) \quad P = \frac{20,000}{120,000} = 0.167$$

Point 2:

EMV(small plant) = EMV(large plant)

$$\$120,000P - \$20,000(1-P) = \$380,000P - \$180,000(1-P)$$
$$P = \frac{160,000}{260,000} = 0.615$$



# SENSITIVITY ANALYSIS

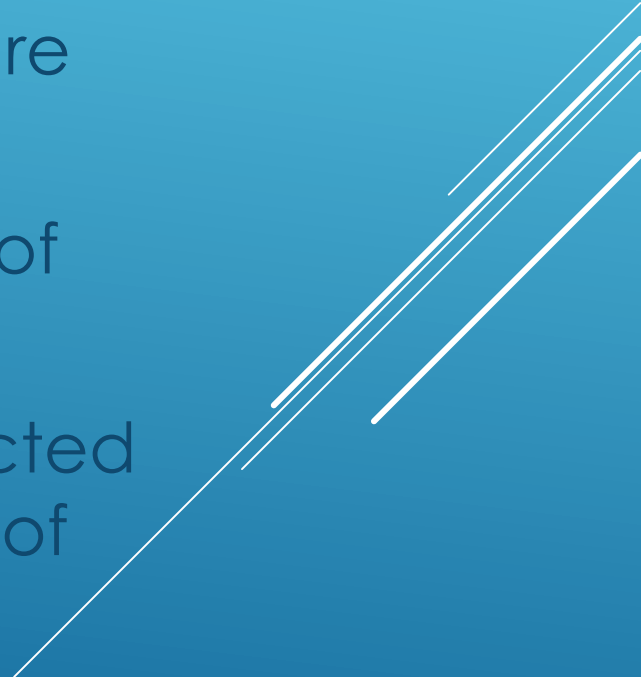


Figure 3.1

# DECISION TREES

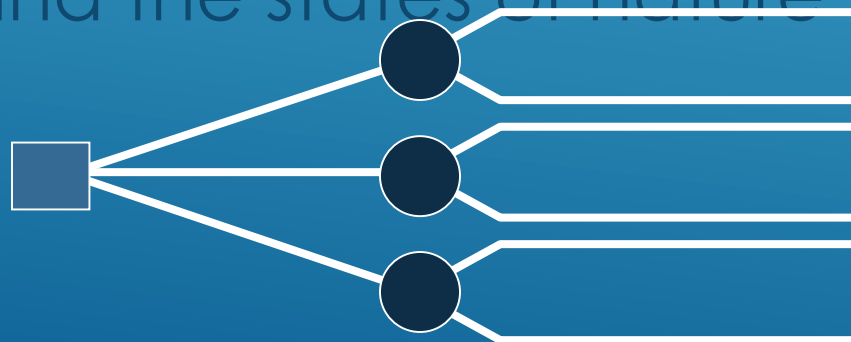
- ▶ Any problem that can be presented in a decision table can also be graphically represented in a *decision tree*
- ▶ Decision trees are most beneficial when a sequence of decisions must be made
- ▶ All decision trees contain *decision points* or *nodes* and *state-of-nature points* or *nodes*
  - ▶ A decision node from which one of several alternatives may be chosen
  - ▶ A state-of-nature node out of which one state of nature will occur

# FIVE STEPS TO DECISION TREE ANALYSIS

1. Define the problem
  2. Structure or draw the decision tree
  3. Assign probabilities to the states of nature
  4. Estimate payoffs for each possible combination of alternatives and states of nature
  5. Solve the problem by computing expected monetary values (EMVs) for each state of nature node
- 
- A series of three parallel white diagonal lines are located in the bottom right corner of the slide, extending from the middle of the right edge towards the bottom left.

# STRUCTURE OF DECISION TREES

- ▶ Trees start from left to right
- ▶ Represent decisions and outcomes in sequential order
- ▶ Squares represent decision nodes
- ▶ Circles represent states of nature nodes
- ▶ Lines or branches connect the decisions nodes and the states of nature



# THOMPSON'S DECISION TREE

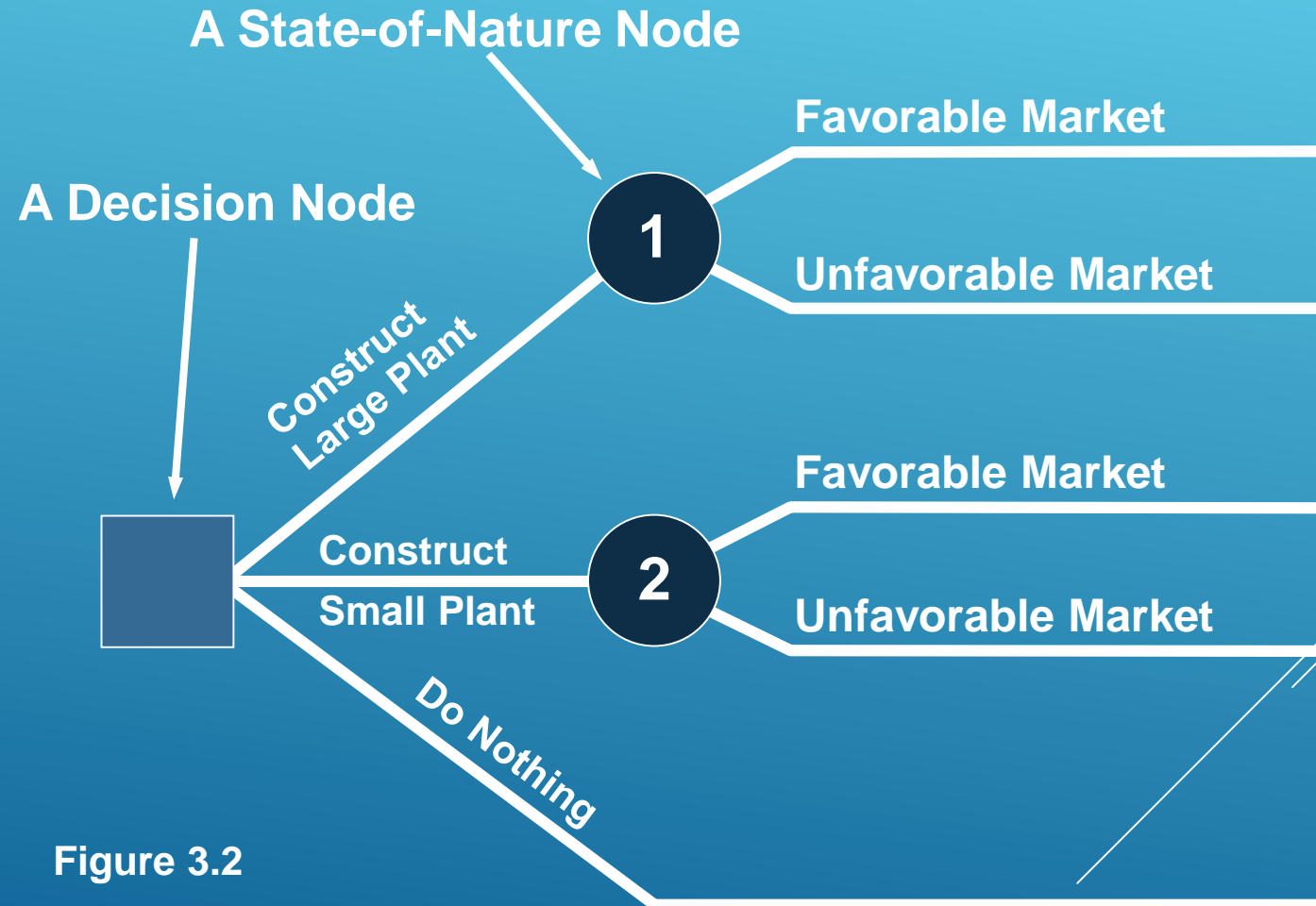


Figure 3.2

# THOMPSON'S DECISION TREE

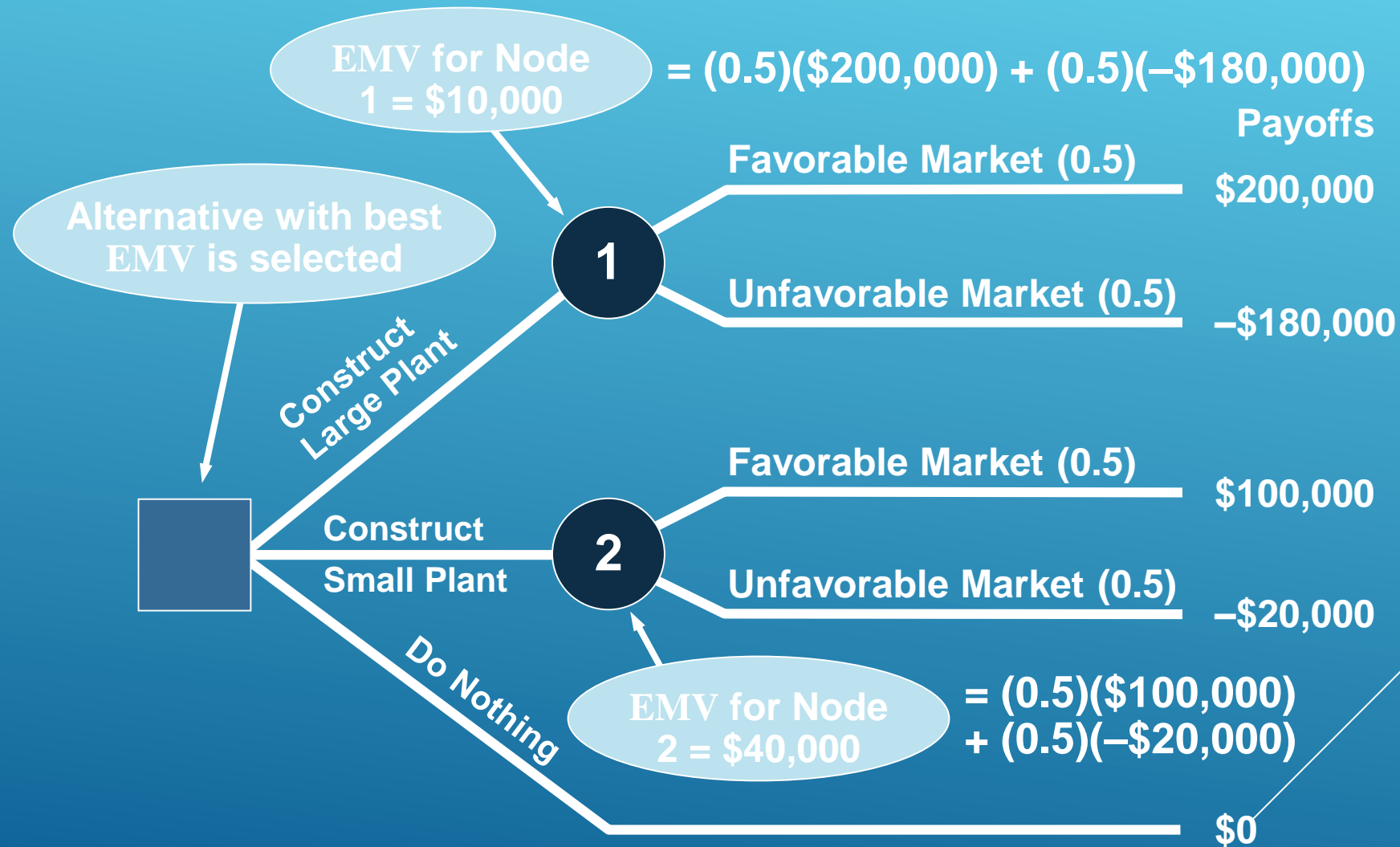


Figure 3.3

# THOMPSON'S COMPLEX DECISION TREE USING SAMPLE INFORMATION

- ▶ Thompson Lumber has two decisions to make, with the second decision dependent upon the outcome of the first
  - ▶ First, whether or not to conduct their own marketing survey, at a cost of \$10,000, to help them decide which alternative to pursue (large, small or no plant)
    - ▶ The survey does not provide perfect information
  - ▶ Then, to decide which type of plant to build
    - ▶ Note that the \$10,000 cost was subtracted from each of the first 10 branches. The, \$190,000 payoff was originally \$200,000 and the \$-10,000 was originally \$0.

# THOMPSON'S COMPLEX DECISION TREE

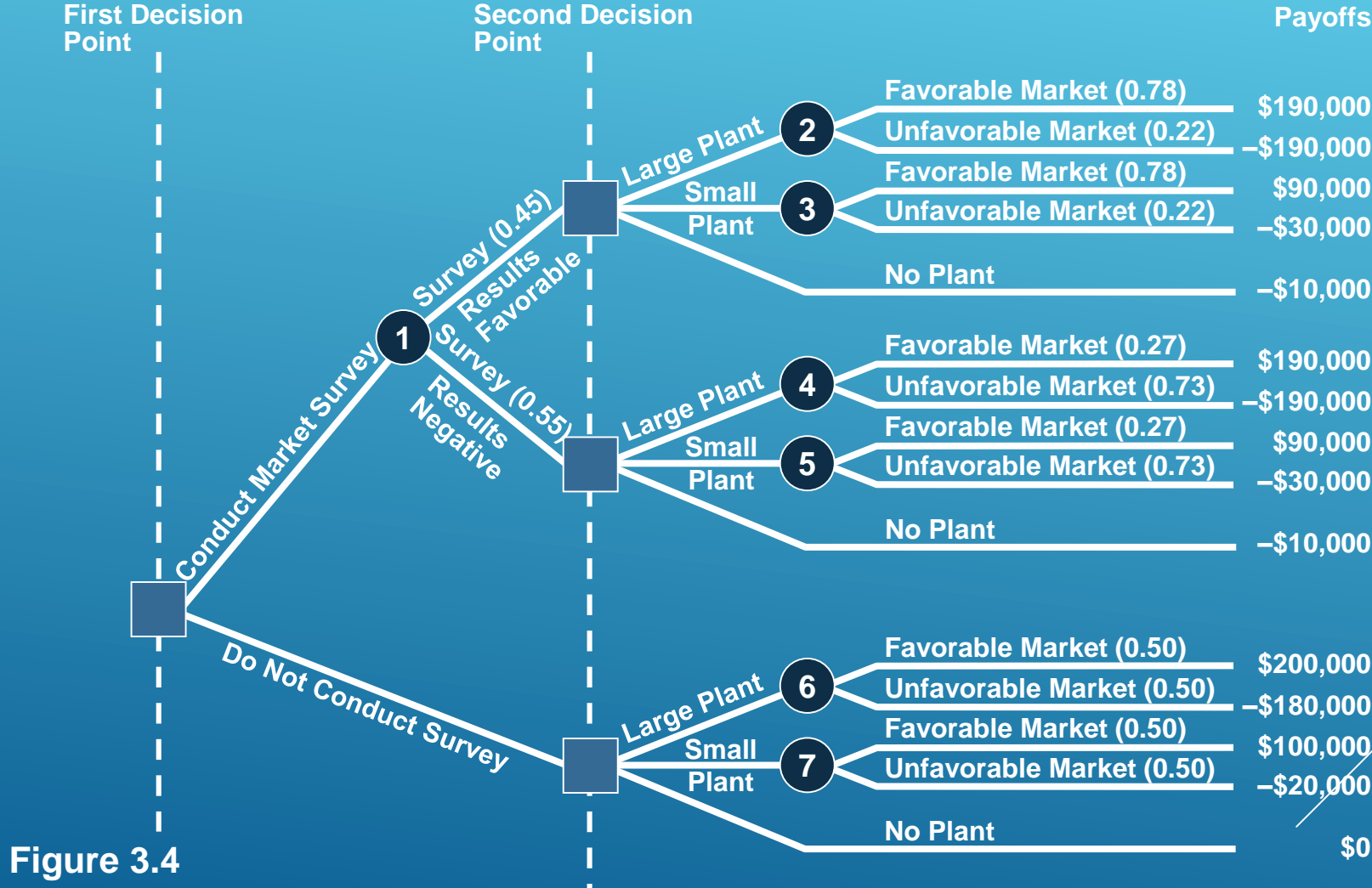


Figure 3.4



# THOMPSON'S COMPLEX DECISION TREE

## 1. Given favorable survey results

(market favorable for sheds),

$$\begin{aligned}\text{EMV}(\text{node 2}) &= \text{EMV}(\text{large plant} \mid \text{positive survey}) \\ &= (0.78)(\$190,000) + (0.22)(-\$190,000) = \$106,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 3}) &= \text{EMV}(\text{small plant} \mid \text{positive survey}) \\ &= (0.78)(\$90,000) + (0.22)(-\$30,000) = \$63,600\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

## 2. Given negative survey results,

$$\begin{aligned}\text{EMV}(\text{node 4}) &= \text{EMV}(\text{large plant} \mid \text{negative survey}) \\ &= (0.27)(\$190,000) + (0.73)(-\$190,000) = -\$87,400\end{aligned}$$

$$\begin{aligned}\text{EMV}(\text{node 5}) &= \text{EMV}(\text{small plant} \mid \text{negative survey}) \\ &= (0.27)(\$90,000) + (0.73)(-\$30,000) = \$2,400\end{aligned}$$

$$\text{EMV for no plant} = -\$10,000$$

# THOMPSON'S COMPLEX DECISION TREE

3. Compute the expected value of the market survey,  
$$\begin{aligned}\text{EMV}(\text{node 1}) &= \text{EMV}(\text{conduct survey}) \\ &= (0.45)(\$106,400) + (0.55)(\$2,400) \\ &= \$47,880 + \$1,320 = \$49,200\end{aligned}$$
4. If the market survey is not conducted,  
$$\begin{aligned}\text{EMV}(\text{node 6}) &= \text{EMV}(\text{large plant}) \\ &= (0.50)(\$200,000) + (0.50)(-\$180,000) = \$10,000 \\ \text{EMV}(\text{node 7}) &= \text{EMV}(\text{small plant}) \\ &= (0.50)(\$100,000) + (0.50)(-\$20,000) = \$40,000 \\ \text{EMV for no plant} &= \$0\end{aligned}$$
5. Best choice is to seek marketing information

# THOMPSON'S COMPLEX DECISION TREE

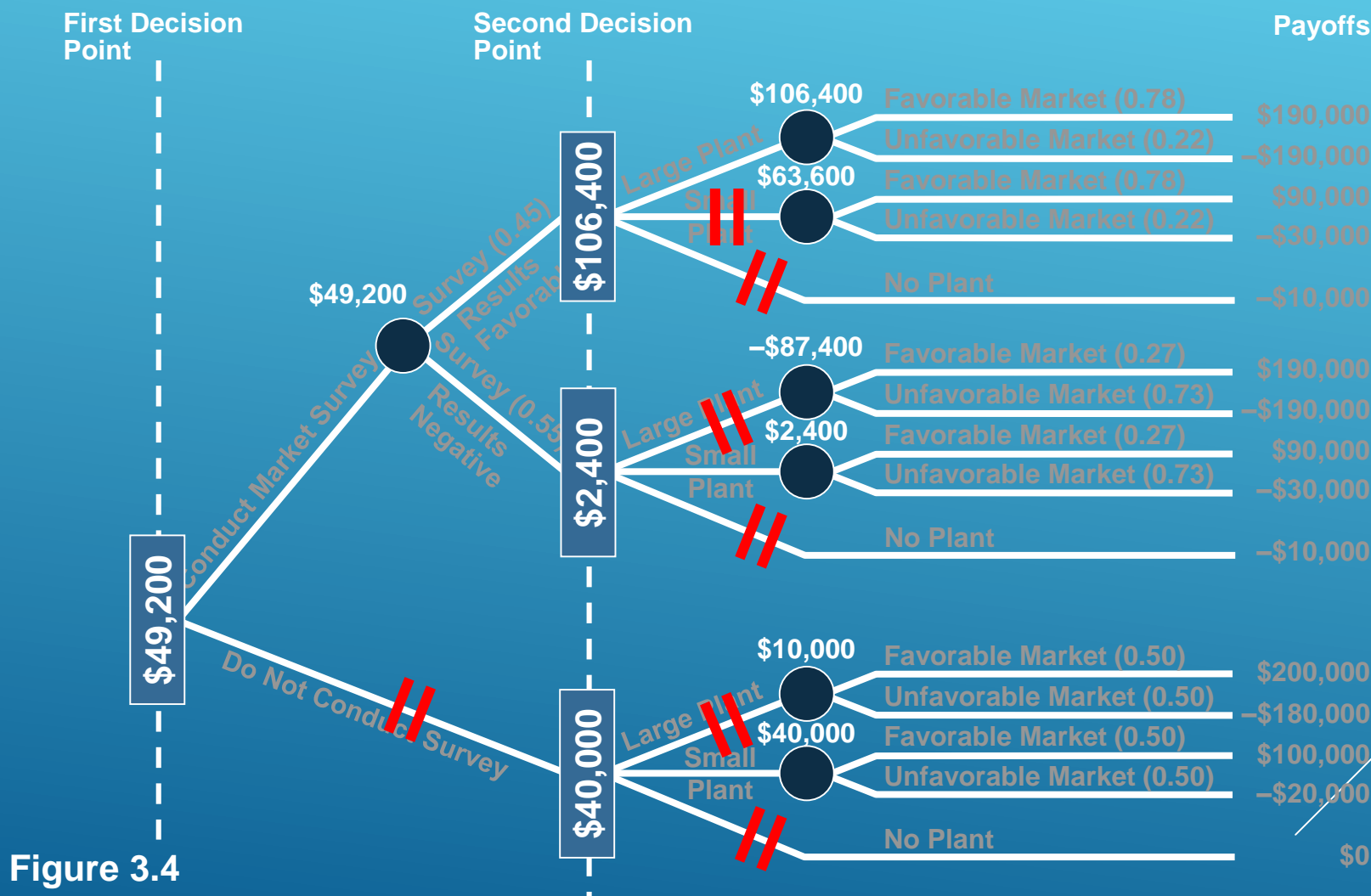
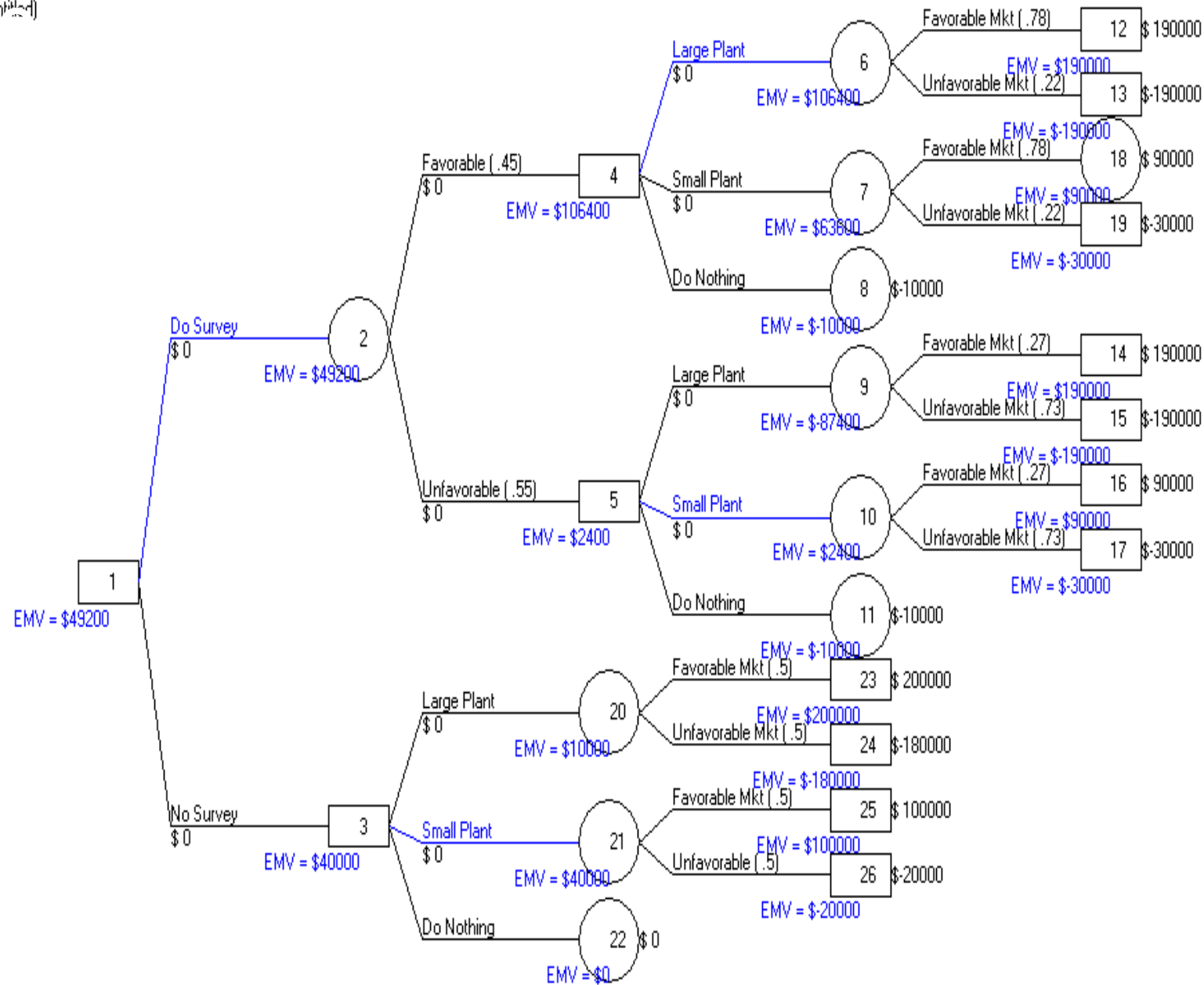


Figure 3.4

(unlabeled)



# EXPECTED VALUE OF SAMPLE INFORMATION

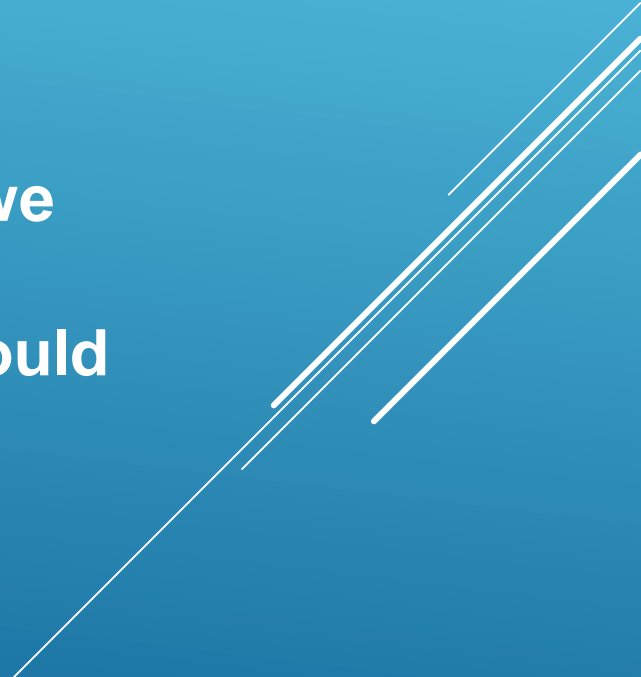
- ▶ Thompson wants to know the actual value of doing the survey

$$\begin{aligned} \text{EVSI} &= \left[ \begin{array}{c} \text{Expected value} \\ \text{with sample} \\ \text{information, assuming} \\ \text{no cost to gather it} \end{array} \right] - \left[ \begin{array}{c} \text{Expected value} \\ \text{of best decision} \\ \text{without sample} \\ \text{information} \end{array} \right] \\ &= (\text{EV with sample information} + \text{cost}) \\ &\quad - (\text{EV without sample information}) \end{aligned}$$

$$\text{EVSI} = (\$49,200 + \$10,000) - \$40,000 = \$19,200$$

Thompson could have paid up to \$19,200 for a market study and still come out ahead since the survey actually costs \$10,000

# SENSITIVITY ANALYSIS

- How sensitive are the decisions to changes in the probabilities?
    - How sensitive is our decision to the probability of a favorable survey result?
    - That is, if the probability of a favorable result ( $p = .45$ ) were to change, would we make the same decision?
    - How much could it change before we would make a different decision?
- 
- A series of three parallel white diagonal lines extending from the bottom right towards the top right of the slide.

# SENSITIVITY ANALYSIS

$p$  = probability of a favorable survey result

$(1 - p)$  = probability of a negative survey result

$$\begin{aligned}\text{EMV}(\text{node 1}) &= (\$106,400)p + (\$2,400)(1 - p) \\ &= \$104,000p + \$2,400\end{aligned}$$

We are indifferent when the EMV of node 1 is the same as the EMV of not conducting the survey, \$40,000

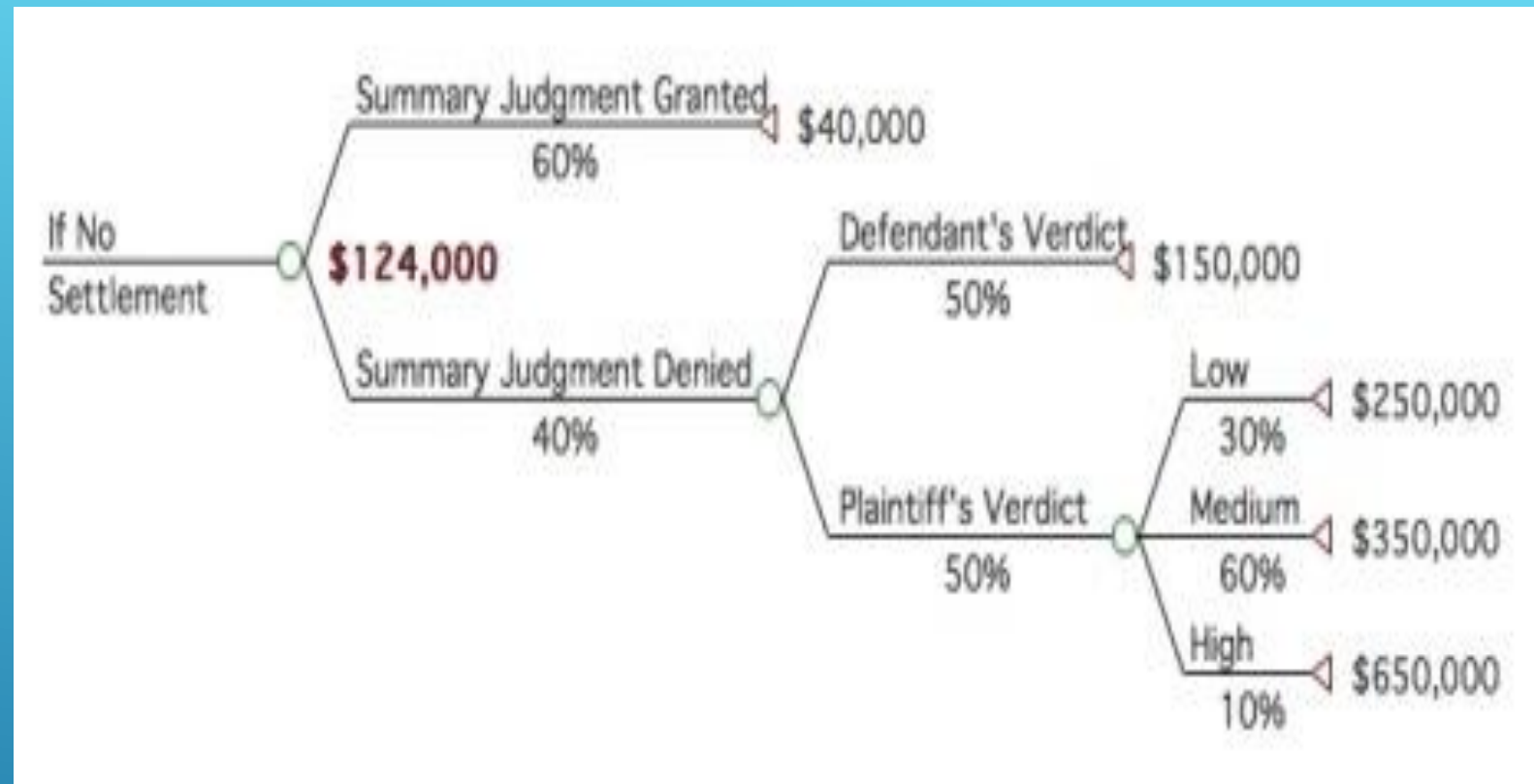
$$\$104,000p + \$2,400 = \$40,000$$

$$\$104,000p = \$37,600$$

$$p = \$37,600 / \$104,000 = 0.36$$

$p > .36$ , the decision will stay the same

$p < .36$ , do not conduct survey



# DECISION TREE ANALYSIS IN LITIGATION

A more complex case [here](#)



# UTILITY THEORY

- ▶ Monetary value is not always a true indicator of the overall value of the result of a decision
- ▶ The overall value of a decision is called *utility*
- ▶ Rational people make decisions to maximize their utility
- ▶ *Should you buy collision insurance on a new, expensive car? Buying the insurance removes a gamble but usually the premium is greater than the expected cost of damage.*
- ▶ *Let's say you were offered \$2,000,000 right now on a chance to win \$5,000,000. The \$5,000,000 is won only if you flip a fair coin and get tails. If you get heads you lose and get \$0. What would you do? Why? What does EMV tell you to do?*
  - ▶ *What if the dollar amounts were \$2,000 guaranteed and \$5,000 if you get tails?*

# UTILITY THEORY

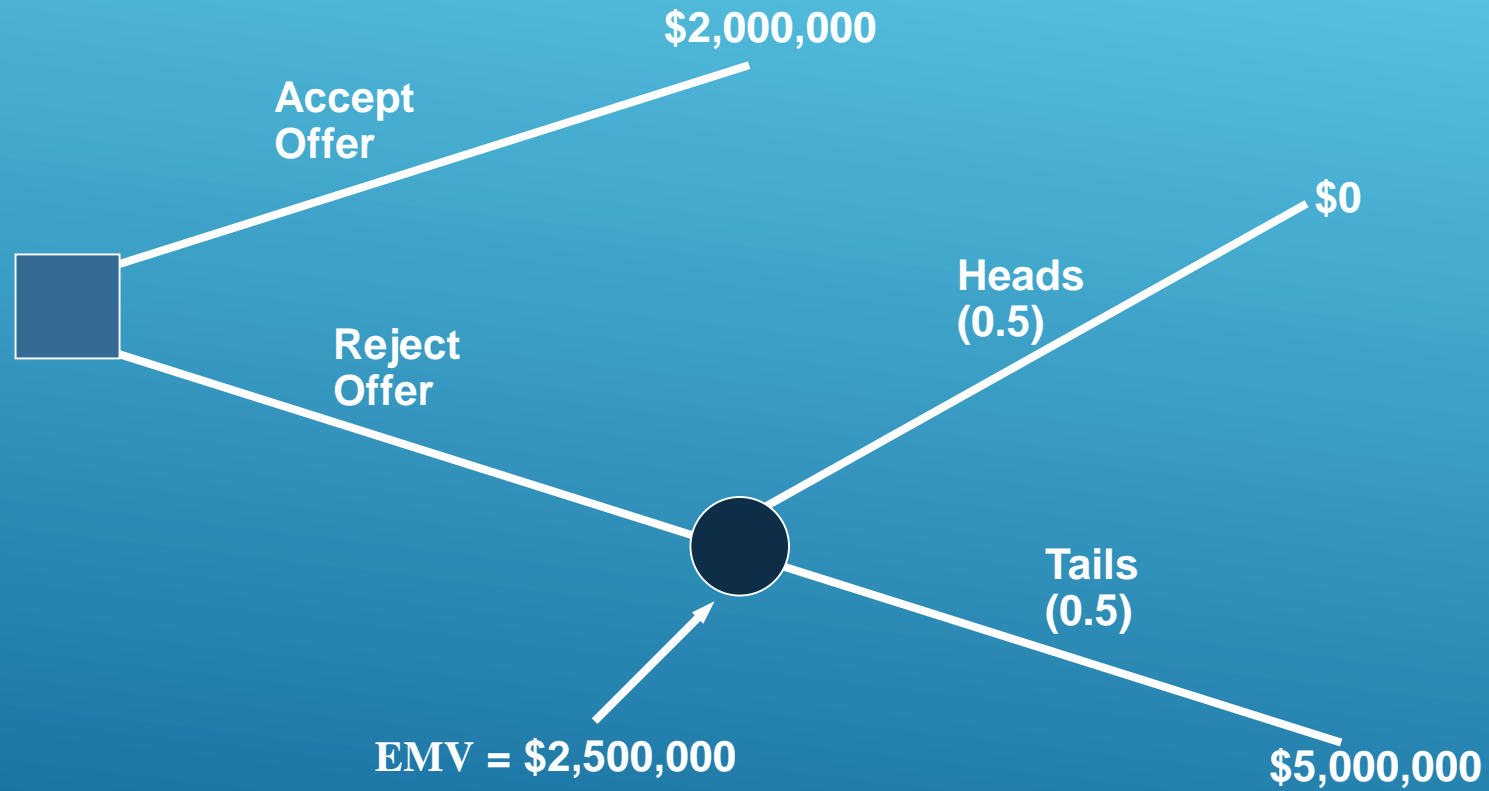


Figure 3.6

# UTILITY THEORY

- ▶ *Utility theory* allows you to incorporate your own attitudes toward risk
  - ▶ Many people would take \$2,000,000 (or even less) rather than flip the coin even though the EMV says otherwise.
  - ▶ A person's utility function could change over time. \$100 as a student vs \$100 as the CEO of a company

# UTILITY THEORY

- ▶ Assign utility values to each monetary value in a given situation, completely subjective
- ▶ *Utility assessment* assigns the worst outcome a utility of 0, and the best outcome, a utility of 1
- ▶ A *standard gamble* is used to determine utility values
  - $p$  is the probability of obtaining the best outcome and  $(1-p)$  the worst outcome
  - Assessing the utility of any other outcome involves determining the probability which makes you indifferent between alternative 1 (gamble between the best and worst outcome) and alternative 2 (obtaining the other outcome for sure)
  - When you are *indifferent*, between alternatives 1 and 2, the expected utilities for these two alternatives must be equal.

# STANDARD GAMBLE

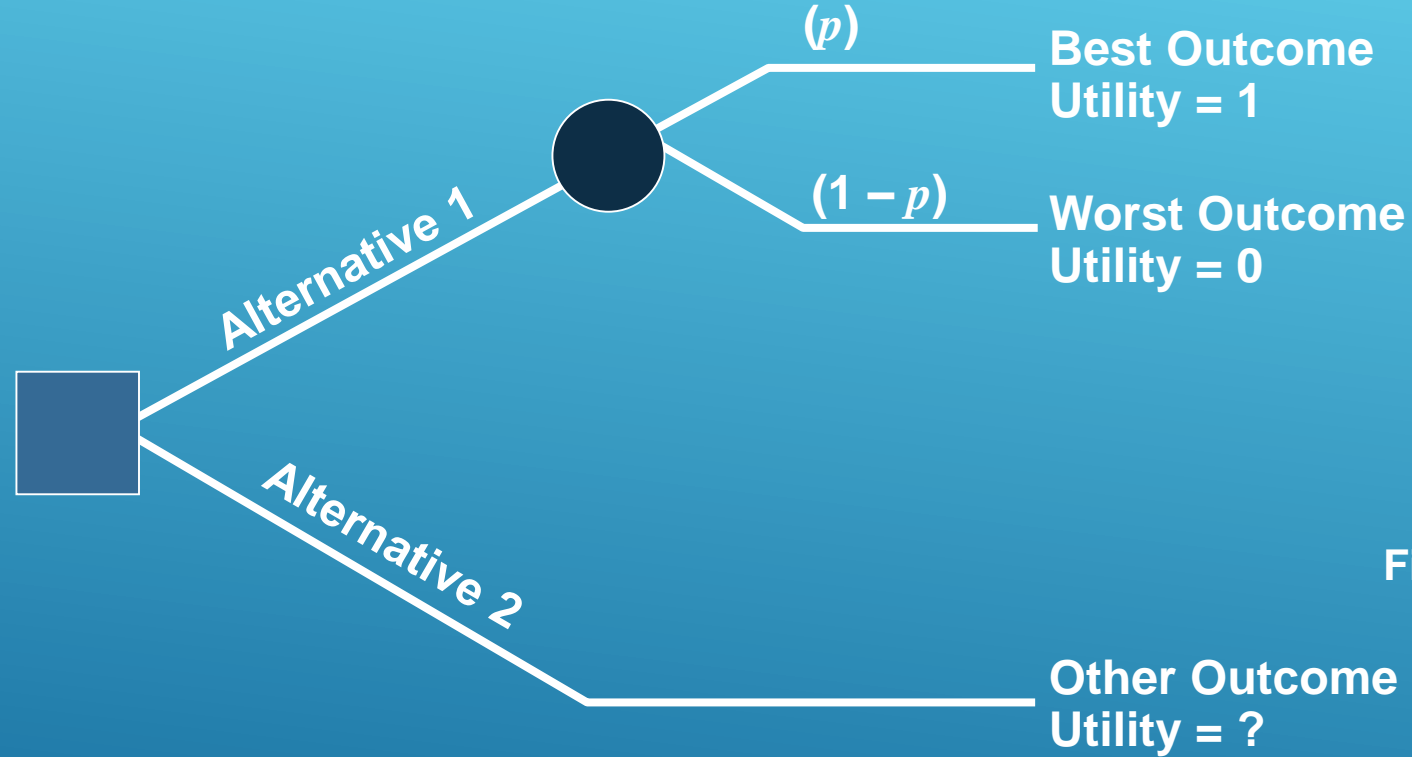


Figure 3.7

*Expected utility of alternative 2 = Expected utility of alternative 1*

*Utility of other outcome = ( $p$ )(utility of best outcome, which is 1)  
+ ( $1 - p$ )(utility of the worst outcome, which is 0)*

*Utility of other outcome = ( $p$ )(1) + ( $1 - p$ )(0) =  $p$*

# STANDARD GAMBLE

- ▶ You have a 50% chance of getting \$0 and a 50% chance of getting \$50,000.
  - ▶ The EMV of this gamble is \$25,000
- ▶ What is the minimum guaranteed amount that you will accept in order to walk away from this gamble?
  - ▶ Or, what is the minimum amount that will make you indifferent between alternative 1 and alternative 2?
- ▶ Suppose you are ready to accept a guaranteed payoff of \$15,000 to avoid the risk associated with the gamble.
  - ▶ From a utility perspective (not EMV), the expected value between \$0 and \$50,000 is only \$15,000 and not \$25,000
  - ▶  $U(\$15,000) = U(\$0) \times .5 + U(\$50,000) \times .5 = 0 \times .5 + 1 \times .5 = .5$

# STANDARD GAMBLE

- ▶ Another way to look characterize a person's risk is to compute the risk premium
  - ▶ Risk premium = (EMV of gamble) – (Certainty equivalent)
  - ▶ This represents how much a person is willing to give up in order to avoid the risk associated with a gamble
- ▶ A person that is more risk averse will be willing to give up an even larger amount to avoid uncertainty
- ▶ A risk taker will insist on getting a certainty equivalent that is greater than the EMV in order to walk away from a gamble
  - ▶ Has a negative risk premium
- ▶ A person who is risk neutral will always specify a certainty equivalent that is exactly equal to the EMV

# INVESTMENT EXAMPLE

- ▶ Jane Dickson wants to construct a utility curve revealing her preference for money between \$0 and \$10,000
- ▶ A utility curve plots the utility value versus the monetary value
- ▶ An investment in a bank will result in \$5,000
- ▶ An investment in real estate will result in \$0 or \$10,000
- ▶ Unless there is an 80% chance of getting \$10,000 from the real estate deal, Jane would prefer to have her money in the bank
- ▶ So if  $p = 0.80$ , Jane is indifferent between the bank or the real estate investment



# INVESTMENT EXAMPLE

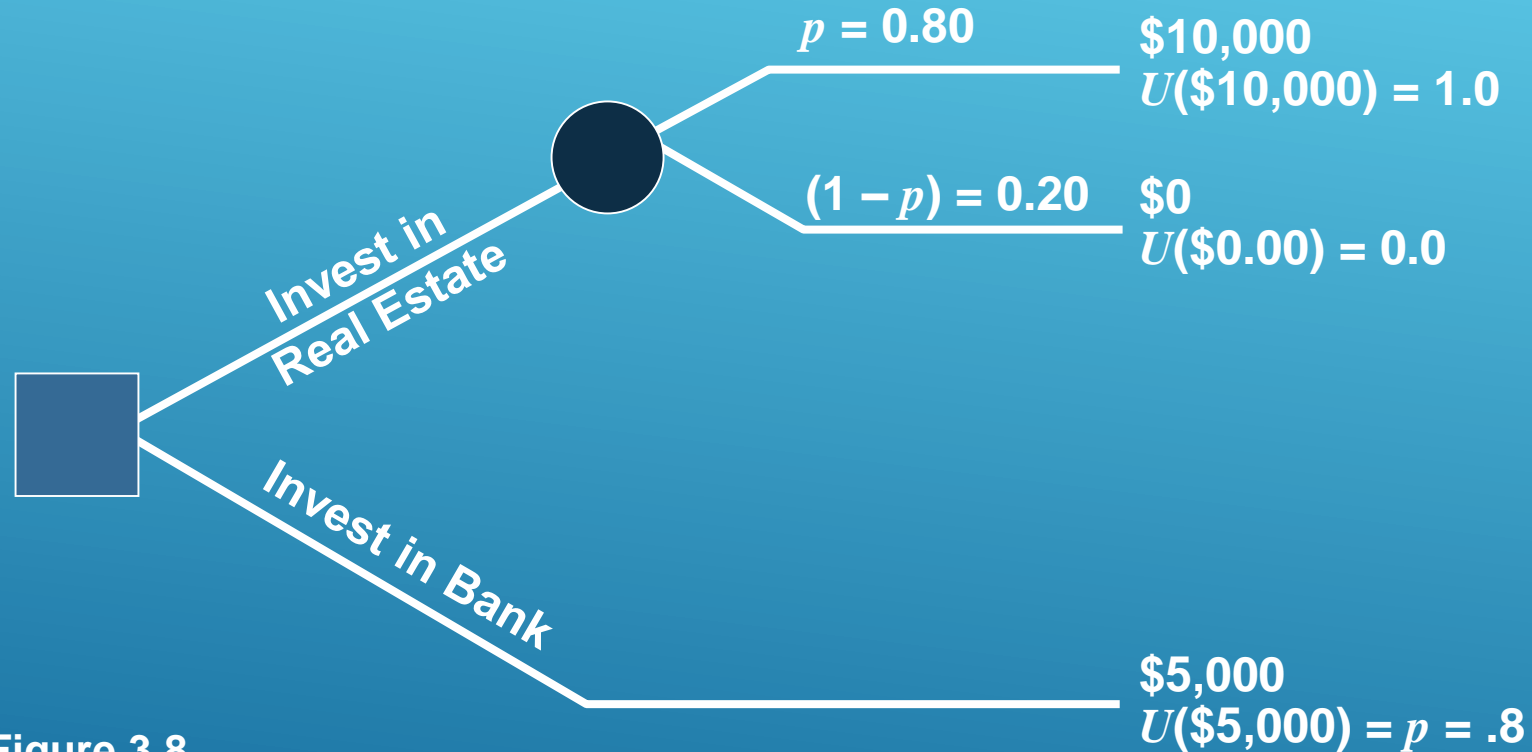


Figure 3.8

$$\begin{aligned}\text{Utility for \$5,000} &= U(\$5,000) = pU(\$10,000) + (1 - p)U(\$0) \\ &= (0.8)(1) + (0.2)(0) = 0.8\end{aligned}$$

# INVESTMENT EXAMPLE

- We can assess other utility values in the same way
- For Jane these are

Utility for \$7,000 = 0.90

Utility for \$3,000 = 0.50

- There must be a 90% chance of getting \$10,000, otherwise she would prefer the \$7,000 for sure
- Using the three utilities for different dollar amounts, she can construct a utility curve

# UTILITY CURVE

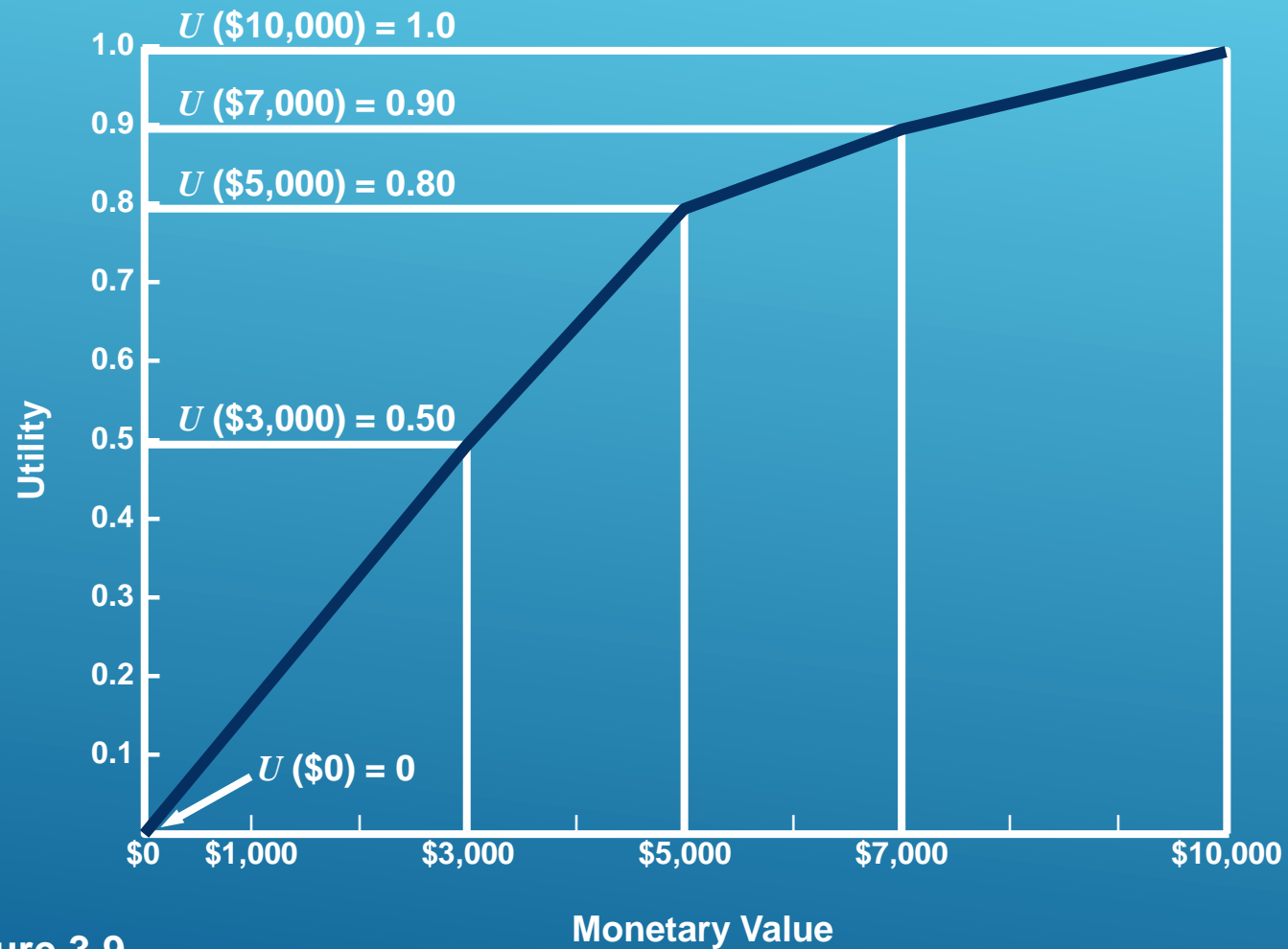


Figure 3.9

# UTILITY CURVE

- **Jane's utility curve is typical of a risk avoider**
  - A risk avoider gets less utility from greater risk
  - Avoids situations where high losses might occur
  - As monetary value increases, the utility curve increases at a slower rate
- A risk seeker gets more utility from greater risk
- As monetary value increases, the utility curve increases at a faster rate
- Someone who is indifferent will have a linear utility curve

# UTILITY CURVE

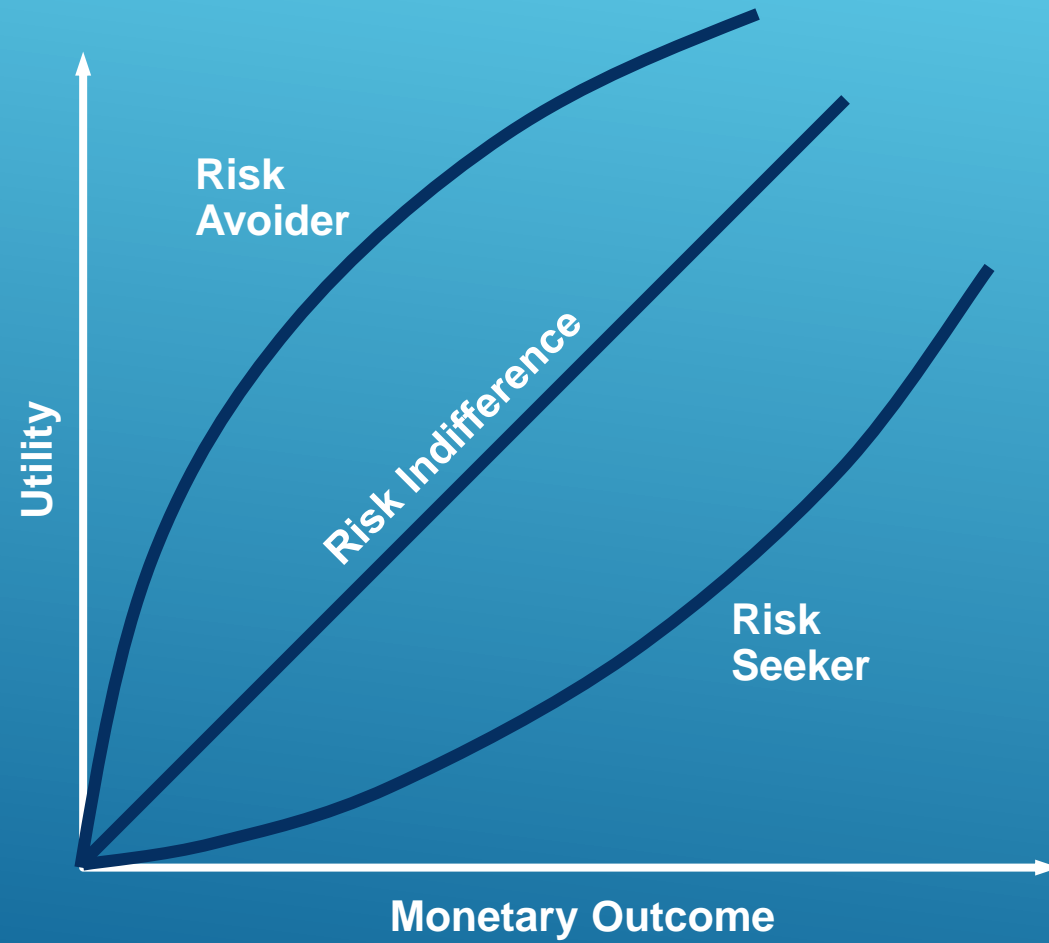



Figure 3.10

# UTILITY AS A DECISION-MAKING CRITERIA

- Once a utility curve has been developed it can be used in making decisions
  - Replace monetary outcomes with utility values
  - The expected utility is computed instead of the EMV
- 
- A series of three parallel white diagonal lines in the bottom right corner of the slide, extending from the bottom edge towards the right edge.

# UTILITY AS A DECISION-MAKING CRITERIA

- **Mark Simkin loves to gamble**
- **He plays a game tossing thumbtacks in the air**
- **If the thumbtack lands point up, Mark wins \$10,000**
- **If the thumbtack lands point down, Mark loses \$10,000**
- **Should Mark play the game (alternative 1)?**

# UTILITY AS A DECISION-MAKING CRITERIA



Figure 3.11



# UTILITY AS A DECISION-MAKING CRITERIA

## ■ Step 1– Define Mark's utilities

$$U (-\$10,000) = 0.05$$

$$U (\$0) = 0.15$$

$$U (\$10,000) = 0.30$$

## ■ Step 2 – Replace monetary values with utility values

$$\begin{aligned} E(\text{alternative 1: play the game}) &= (0.45)(0.30) + (0.55)(0.05) \\ &= 0.135 + 0.027 = 0.162 \end{aligned}$$

$$E(\text{alternative 2: don't play the game}) = 0.15$$

# UTILITY AS A DECISION-MAKING CRITERIA

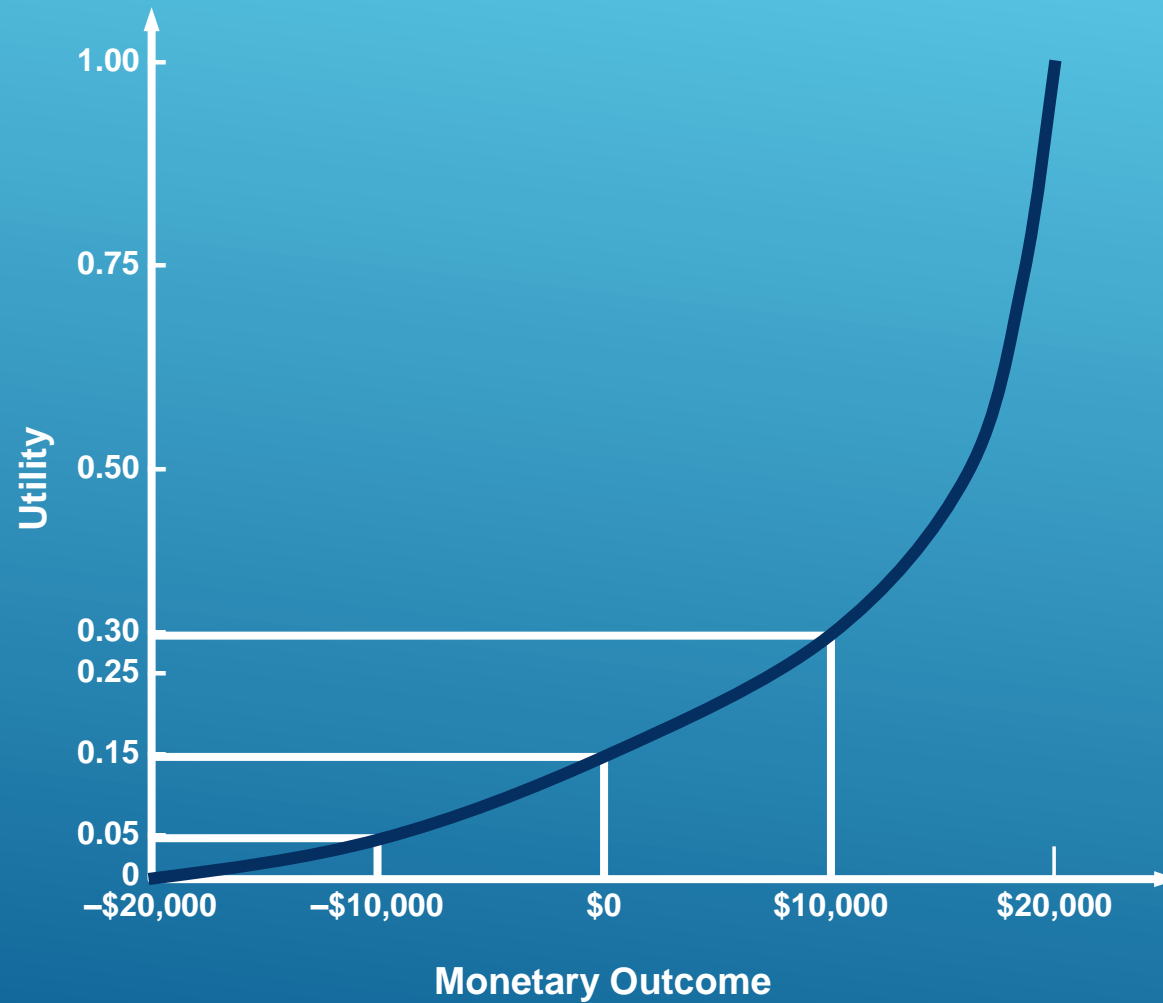


Figure 3.12

# UTILITY AS A DECISION-MAKING CRITERIA

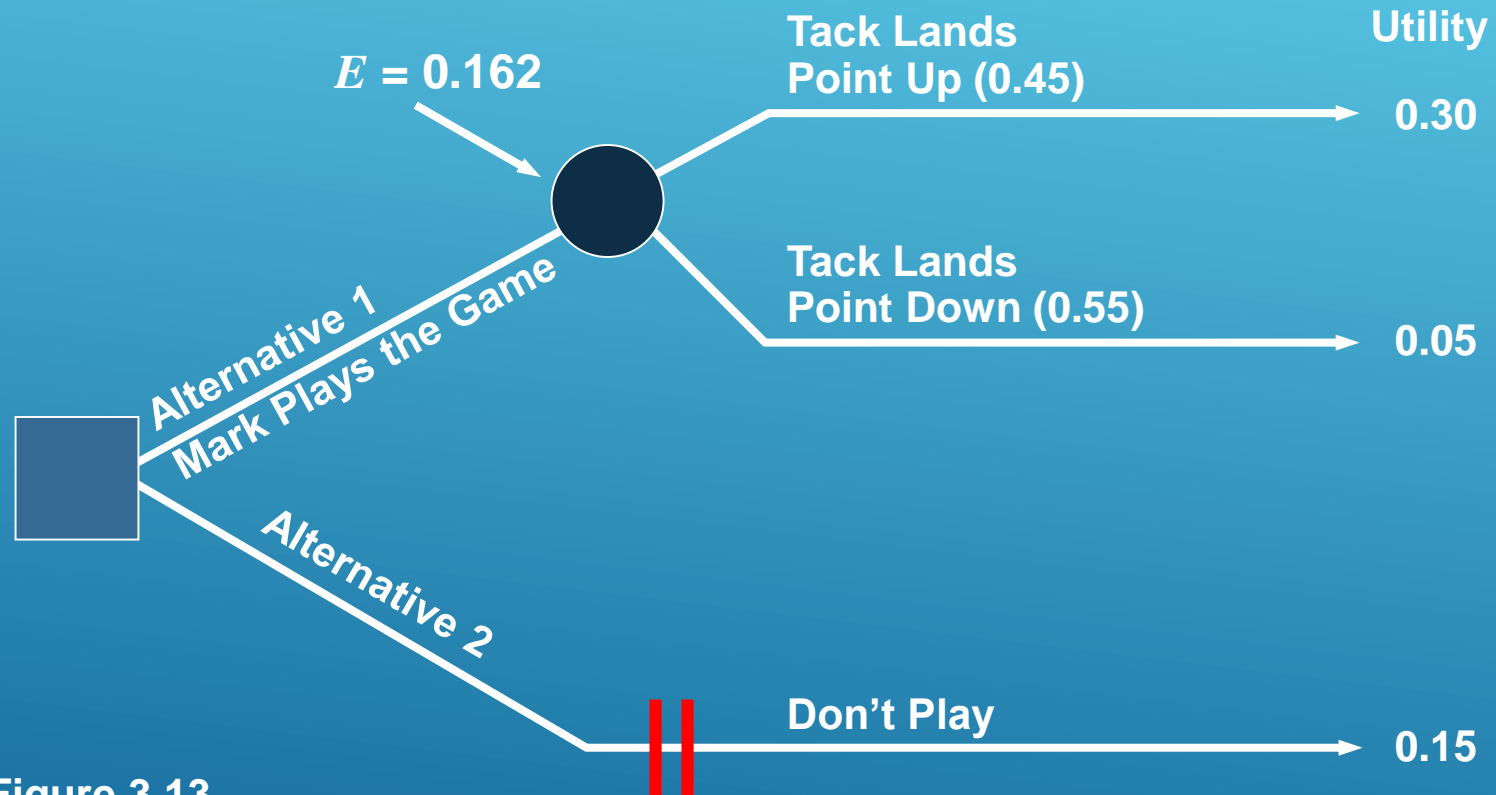


Figure 3.13

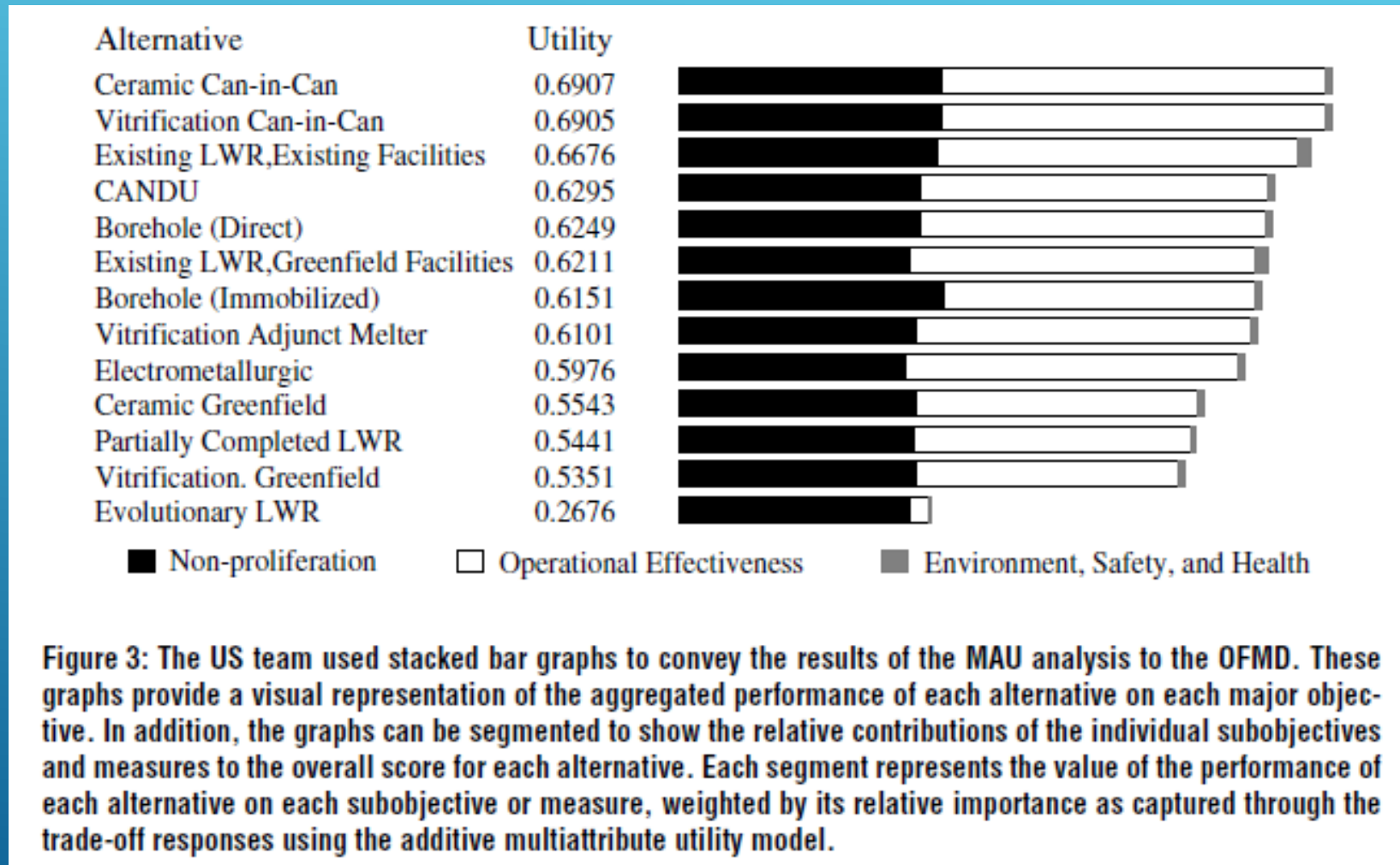
# UTILITY AS A DECISION-MAKING CRITERIA

- A life insurance company sells term life insurance.
- If the policy holder dies during the term of the policy the company pays \$100,000, otherwise \$0
- Based on actuarial tables, the probability of a person dying during the next year is .001
- The cost of the policy is \$200
- Based on EMV, should the individual buy the policy?
- How does utility theory explain why a person would buy the policy?

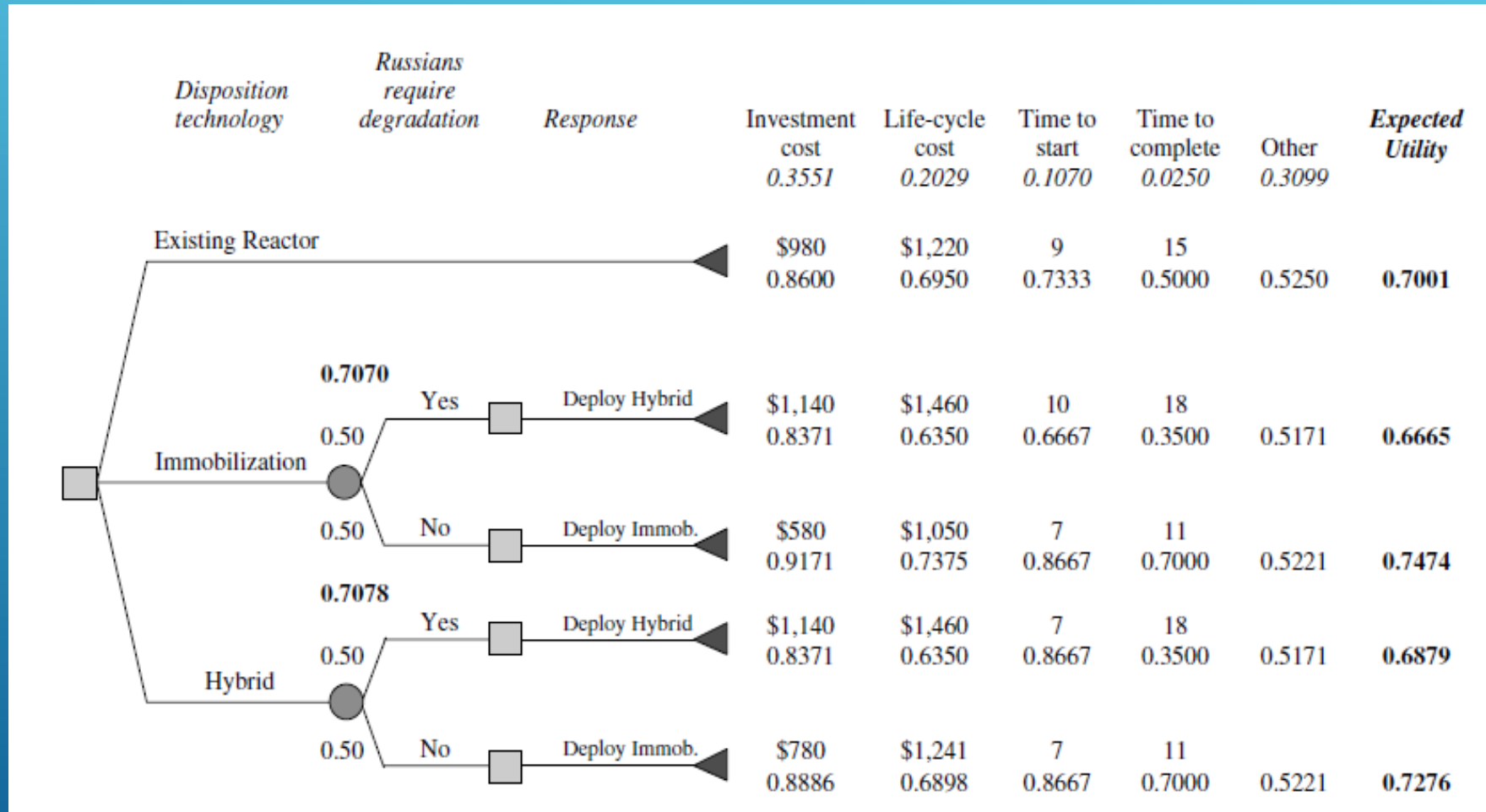
# MULTI-ATTRIBUTE UTILITY (MAU) MODELS

- Multi-attribute utility (MAU) models are mathematical tools for evaluating and comparing alternatives to assist in decision making about complex alternatives, especially when groups are involved.
- They are designed to answer the question, "What's the best choice?"
- The models allow you to assign scores to alternative choices in a decision situation where the alternatives can be identified and analyzed.
- They also allow you to explore the consequences of different ways of evaluating the choices.
- The models are based on the assumption that the apparent desirability of a particular alternative depends on how its attributes are viewed.
  - For example, if you're shopping for a new car, you will prefer one over another based on what you think is important, such as price, reliability, safety ratings, fuel economy, and style.

# MAU MODELS FOR PLUTONIUM DISPOSITION



# MAU MODELS FOR PLUTONIUM DISPOSITION





# MAU MODELS FOR PLUTONIUM DISPOSITION

**Figure 4:** The measure on Russian cooperation was replaced with a probability distribution on whether Russia required degradation vis-à-vis reactor technology. The numbers (e.g., 0.3551) immediately under the column headings are the values of the weights on the corresponding measures, while the numbers immediately under the values for each alternative (e.g., 0.8600) are the corresponding scores from the single-attribute utility functions for the measures. The numbers in the expected utility column are the expected utilities of the alternatives determined by multiplying each weight times the corresponding score for a measure and summing the results. The analysis takes the form of a decision tree with the following logic: first, the US must select a disposition strategy without knowing Russia's future stand on the isotopic degradation of the plutonium; second, after the US announces its decision, Russia will announce its official policy; finally, the US will have the option of reacting to the Russian announcement. This decision tree provided the OFMD with information regarding the attractiveness of each alternative as a function of the probability that Russian policy required degradation. The analysis revealed that each of the three alternatives could be the most preferred; this emphasized the value of the flexibility provided by the hybrid approach.



# MAU MODELS FOR PLUTONIUM DISPOSITION

This operations research application had the following impacts:

- The MAU model identified the information the OFMD required to evaluate the 13 plutonium-disposition alternatives and structured the data collection and analysis effort.

- The use of 37 performance measures in the MAU model ensured that the discussions did not focus only on the strengths of some alternatives or only on their weaknesses. The OFMD team managed to avoid an emotional dispute over the alternatives and to maintain a reasoned discussion with different points of view revealed clearly and with a balanced perspective on the alternatives.

- The presentation of the preliminary results of the MAU analysis led to the identification of inconsistencies in data collection and caused the OFMD team to audit and standardize the measures of attribute performance. In addition, the analysis highlighted deficiencies in some of the alternatives, which helped the science teams to modify some of the alternatives to improve their performance measures on key attributes.

- The OFMD recommended two of the alternatives ranked highest by the final MAU analysis for parallel development. The MAU analysis provided the only quantification of the benefit of parallel development. The DOE adopted this hybrid approach (DOE ROD 97), and the US then reacted to subsequent Russian policy decisions by selecting one of these options for further development.

The selected option calls for disposal of plutonium by fabricating it into mixed oxide (MOX) fuel for irradiation in existing, commercial nuclear reactors.

# MAU MODELS FOR PLUTONIUM DISPOSITION

The Russian study had the following impacts:

- Prior to implementing the Russian MAU model, the predominant view in Russia was to use their excess-weapons plutonium as a fuel source in fast nuclear reactors of an advanced design planned for future construction. The Russian MAU analysis based on consideration of financial and technical support from the rest of the world favored alternatives in which the plutonium would be fabricated into MOX fuel and irradiated in existing Russian nuclear reactors with a shorter time schedule and an estimated cost on the order of \$2 to \$3 billion.

- The Russian scientists presented the recommendations of the Russian version of the MAU model to MINATOM, which took these results into account in its major policy statements.

- The MAU analysis also highlighted the desirability of parallelism between US and Russian plutonium-disposition technologies. The Russians have decided to replicate the design of the US MOX facility in Russia, contributing to the synergy in the disposition policies.

